Laboratori Nazionali di Frascati

LNF-62/65 (1962)

A. Fujii: BETA DECAY OF HYPERONS VIA K'-PARTICLE.

Estratto dal: Nuovo Cimento, 25, 618 (1962)

IL NUOVO CIMENTO VOL. XXV, N. 3 1° Agosto 1962

Beta Decay of Hyperons via K′-Particle.

A. FUJII

Laboratori Nazionali di Frascati - Frascati

(ricevuto il 13 Aprile 1962)

Summary. — The β-decay rate of the hyperons and thel eptonic decay rates of the kaon are calculated in an intermediate boson picture, where the intermediary boson is the K′ particle. A numerical estimate of the coupling constants is also made, though of preliminary nature.

A recent experiment (1) confirmed the fact that the observed β-decay rate of the hyperon is an order of magnitude smaller than that predicted by the unrenormalized universal V—A theory (2). This suggests either the universality breaks down or the renormalization effects are of vital importance if the conventional V—A theory still holds in this process. The vector and axial vector weak current between the hyperon and nucleon can be written in a general form, on the basis of covariance,

$$\langle \mathcal{N} | V_\mu | Y \rangle = \overline{u}_{\mathcal{N}} \left[f_1(s)\gamma_\mu + f_2(s)\sigma_{\mu\nu}\frac{k_\nu}{m_{\mathcal{N}}} + f_3(s)\frac{k_\mu}{m_{\mathcal{N}}} \right] u_Y ,$$

$$\langle \mathcal{N} | P_\mu | Y \rangle = \overline{u}_{\mathcal{N}} \left[g_1(s)\gamma_\mu + g_2(s)\sigma_{\mu\nu}\frac{k_\nu}{m_{\mathcal{N}}} + g_3(s)\frac{k_\mu}{m_{\mathcal{N}}} \right] \gamma_5 u_Y ,$$

where the six form factors f and g are functions of the four-square of the momentum transfer k,

$$s = k^2 , \qquad k = P_Y - P_{\mathcal{N}} .$$

---

(1) W. E. HUMPHREY, J. KIRZ, A. H. ROSENFELD, J. LEITNER and Y. I. RHEE: *Phys. Rev. Lett.*, **6**, 478 (1961).

(2) R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958).

Several authors [3-6] made use of the dispersion theoretic approach to compute these form factors. The intermediate states which connect the hyperon-nucleon current and the lepton current must be characterized by charge −1, strangeness −1, angular momentum either 0 or 1. Only two simple states, *i.e.* the single kaon state and the single kaon plus single pion state, were included in the computation so far. Their conclusions may be summarized briefly as follows [6]. In case of the $\Lambda\mathcal{N}K$ relative parity odd and $\Lambda\Sigma$ parity even, the single kaon state contributes only to g_3. The kaon plus pion state makes a contribution to all f's, but those to f_1 and f_2 are negligibly small. However, the form factors f_3 and g_3 have very little effect on the decay rate, because when the accompanying operator $k_\mu/m_{\mathcal{N}}$ is transfered to the lepton current it yields a small factor $m_e/m_{\mathcal{N}}$:

$$k_\mu \bar{u}_e \gamma_\mu (1+\gamma_5) u_\nu = \bar{u}_e\big((P_e\cdot\gamma) + (P_\nu\cdot\gamma)\big)(1+\gamma_5)u_\nu = i m_e \bar{u}_e (1+\gamma_5) u_\nu \,.$$

Thus the decay rate is mainly controlled by the « subtraction constants » $f_1(0)$ and $g_1(0)$, which the theory leaves undetermined. When f_1 and g_1 are normalized such that the case

$$f_1(0) = g_1(0) = 1$$

corresponds to the unrenormalized V—A theory, the decay rate reads, *e.g.* for the Λ-hyperon [7],

$$w(\Lambda_\beta) = 1.43\cdot 10^7\ \text{s}^{-1}\cdot[|f_1(0)|^2 + 2.96|g_1(0)|^2]\,.$$

A further dynamical model is required to find the « renormalized coupling constants » $f_1(0)$ and $g_1(0)$. For the nucleon β-decay the hypothesis of partially conserved axial vector current [8] was remarkably successful. This hypothesis assumes that the divergence of the axial vector nucleon current P'_μ is proportional to the renormalized pion field

$$\partial_\mu P'_\mu = i a_\pi \varphi_\pi\,,$$

---

(3) E. M. FERREIRA: *Nuovo Cimento*, **8**, 359 (1958).
(4) N. CABIBBO and R. GATTO: *Nuovo Cimento*, **13**, 1086 (1959).
(5) D. FLAMM: *Nuovo Cimento*, **16**, 194 (1960).
(6) D. R. HARRINGTON: *Phys. Rev.*, **124**, 1290 (1961).
(7) D. R. HARRINGTON: *Phys. Rev.*, **120**, 1482 (1960).
(8) J. BERNSTEIN, S. FUBINI, M. GELL-MANN and W. THIRRING: *Nuovo Cimento*, **17**, 757 (1960).

where the real constant a_π is determined by the pion decay rate

$$w(\pi^+ \to \mu^+ + \nu) = \frac{G^2}{8\pi m_\pi}\left(\frac{m_\mu}{m_\pi}\right)^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 \cdot a_\pi^2\,,$$

G being the Fermi constant

$$Gm_{\mathcal{N}}^2 \sim 1.0\cdot 10^{-5}.$$

The renormalized axial vector coupling constant G_A for nucleon β-decay is then shown to be related to the renormalized pion-nucleon coupling constant g_π by

$$\lambda = \frac{G_A}{G} = \frac{a_\pi g_\pi}{2m_{\mathcal{N}}\cdot m_\pi^2} \sim 1.2\,.$$

This was actually first derived by GOLDBERGER and TREIMAN by the dispersion theoretic method [9]. We may analogously assume that the divergence of P_μ is proportional to the renormalized kaon field

$$\partial_\mu P_\mu = i a_K \varphi_K\,,$$

where a_K is given in terms of the $K_{\mu 2}$ decay rate

$$w(K^+ \to \mu^+ + \nu) = \frac{G^2}{8\pi m_K}\left(\frac{m_\mu}{m_K}\right)^2\left(1-\frac{m_\mu^2}{m_K^2}\right)^2 a_K^2\,.$$

The corresponding Goldberger-Treiman relation is

$$g_1(0) = \frac{a_K g_K}{(m_Y + m_{\mathcal{N}})\cdot m_K^2}\,,$$

g_K being the renormalized $Y\mathcal{N}K$ coupling constant, hence

$$\frac{g_1(0)}{\lambda} = \frac{a_K}{a_\pi}\cdot\left(\frac{m_\pi}{m_K}\right)^2\cdot\frac{2m_{\mathcal{N}}}{m_Y + m_{\mathcal{N}}}\cdot\frac{g_K}{g_\pi} \sim 0.1\ [10]\,,$$

which alone seems to be too small to fit the experimental decay rate.

To obtain an estimate of the renormalized vector coupling constant $f_1(0)$ we assume no subtraction and introduce a stable vector meson K′, the exper-

---

[9] M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **110**, 1178 (1958).
[10] Y. NAMBU: *Phys. Rev. Lett.*, **4**, 380 (1960).

imentally observed Kπ resonance state at 885 MeV ([11]) and compute $f_1(0)$ in the pole approximation. In the same model the leptonic decay rate of the kaon is called for to find the unknown coupling constants. The K′-particle is assumed to couple strongly to the baryon and meson current by the phenomenological hamiltonians

$$H_1 = ig_Y \bar{\mathcal{N}} \gamma_\mu Y \cdot K'_\mu + \text{h. c.}$$

$$H_2 = g\left(K\frac{\partial \pi^*}{\partial x_\mu} - \pi^* \frac{\partial K}{\partial x_\mu}\right) K'_\mu + \text{h.c.}\,,$$

which are further assumed to be isoscalar. The K′-particle is coupled weakly to the lepton current by

$$H_3 = f(\bar{e}\gamma_\mu(1+\gamma_5)\nu + \bar{\mu}\gamma_\mu(1+\gamma_5)\nu)\cdot K'_\mu + \text{h. c.}$$

The possible coupling of the K′-particle through the «moment» type interaction are ignored. The decay diagram of the hyperon and kaon are shown in Fig. 1. We can immediately see that the decays

$$\Sigma^+ \to n + e^+ + \nu\,, \qquad K^0 \to \pi^+ + e^- + \bar{\nu}\,, \qquad K^0 \to \pi^+ + \mu^- + \bar{\nu}$$

are forbidden in this picture, because , K$^{\pm\prime}$ has strangeness ± 1 and strangeness has to be conserved in a strong vertex. This is in accordance with the

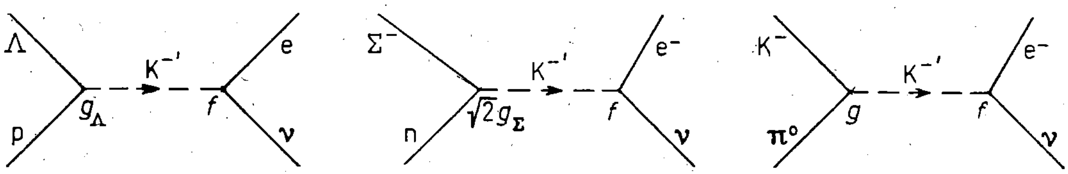

Fig. 1. – Leptonic decay diagrams of the hyperon and kaon via K′-particle.

$\Delta S = \Delta Q$ rule, however this rule seems to have been experimentally disproved recently ([12]). The vector meson K′ actually reproduces the contact interaction of the baryon and lepton currents, because the essential part of the matrix

---

([11]) M. ALSTON, L. W. ALVAREZ, P. EBERHARD, M. L. GOOD, W. GRAZIANO, H. K. TICHO and S. G. WOJCICKI: *Phys. Rev. Lett.*, **6**, 300 (1961).

([12]) R. P. FRY, W. M. POWELL, H. WHITE, M. BALDO-CEOLIN, E. CALIMANI, S. CIAMPOLILLO, O. FABRI, F. FARINI, C. FILIPPI, N. NUZITA, G. MIARI, U. CAMERINI, W. F. FRY and S. NATALI: *Phys. Rev. Lett.*, **8**, 132 (1962).

element becomes

$$\bar{u}_{\mathcal{N}}\gamma_\mu u_Y \cdot \frac{1}{k^2+m_{K'}^2}\cdot\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{m_{K'}^2}\right)\cdot\bar{u}_e\gamma_\nu(1+\gamma_5)u_\nu =$$

$$= \frac{1}{k^2+m_{K'}^2}\cdot\left(\bar{u}_{\mathcal{N}}\gamma_\mu u_Y\cdot\bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu - \frac{(m_Y-m_{\mathcal{N}})m_e}{m_{K'}^2}\bar{u}_{\mathcal{N}}u_Y\cdot\bar{u}_e(1+\gamma_5)u_\nu\right),$$

where the second term is completely negligible due to the small factor $m_e/m_{K'}$. The decay rates are found in a straightforward manner as

$$w(\Lambda_\beta) = \frac{2}{\pi}\cdot\frac{g_\Lambda^2}{4\pi}\cdot\frac{f^2}{4\pi}\cdot m_\Lambda\times 5.42\cdot 10^{-5},$$

$$w(\Sigma_\beta) = \frac{2}{\pi}\cdot\frac{(\sqrt{2}g_\Sigma)^2}{4\pi}\cdot\frac{f^2}{4\pi}\cdot m_\Sigma\times 3.05\cdot 10^{-4},$$

$$w(K_{e3}) = \frac{2}{3\pi}\cdot\frac{g^2}{4\pi}\cdot\frac{f^2}{4\pi}\cdot m_K\times 3.85\cdot 10^{-3},$$

$$w(K_{\mu3}) = \frac{1}{\pi}\cdot\frac{g^2}{4\pi}\cdot\frac{f^2}{4\pi}\cdot m_K\times 2.12\cdot 10^{-3}.$$

In fact a completely analytic integration in phase space is possible for the electron decay modes putting the electron mass zero.

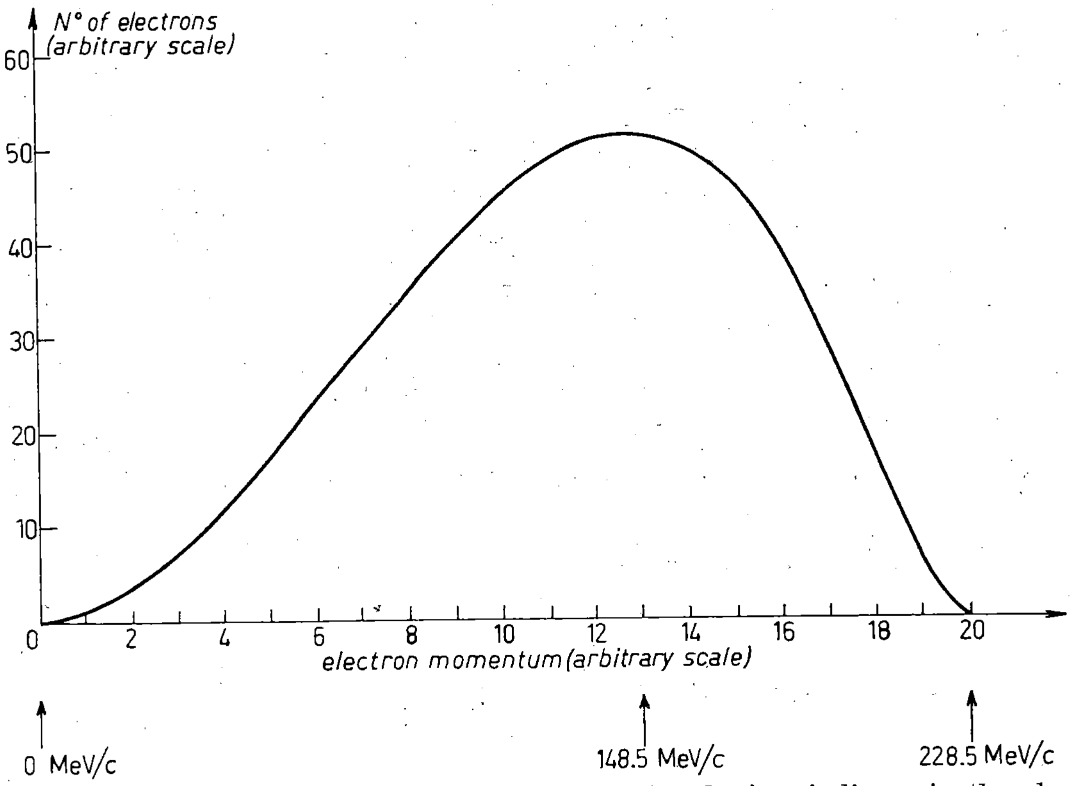

Fig. 2. – Electron spectrum in K_{e3}-decay mode. The abscissa is linear in the electron momentum.

We see immediately that the branching ratio

$$\frac{w(K_{e3})}{w(K_{\mu 3})} \sim 1.2 \, ,$$

is consistent with the experimental value ~ 1. For the purpose of reference in future experiments we draw the lepton spectrum of the kaon decay in Fig. 2 and 3. The muon spectrum in the $K_{\mu 3}$ mode has a peak at muon total

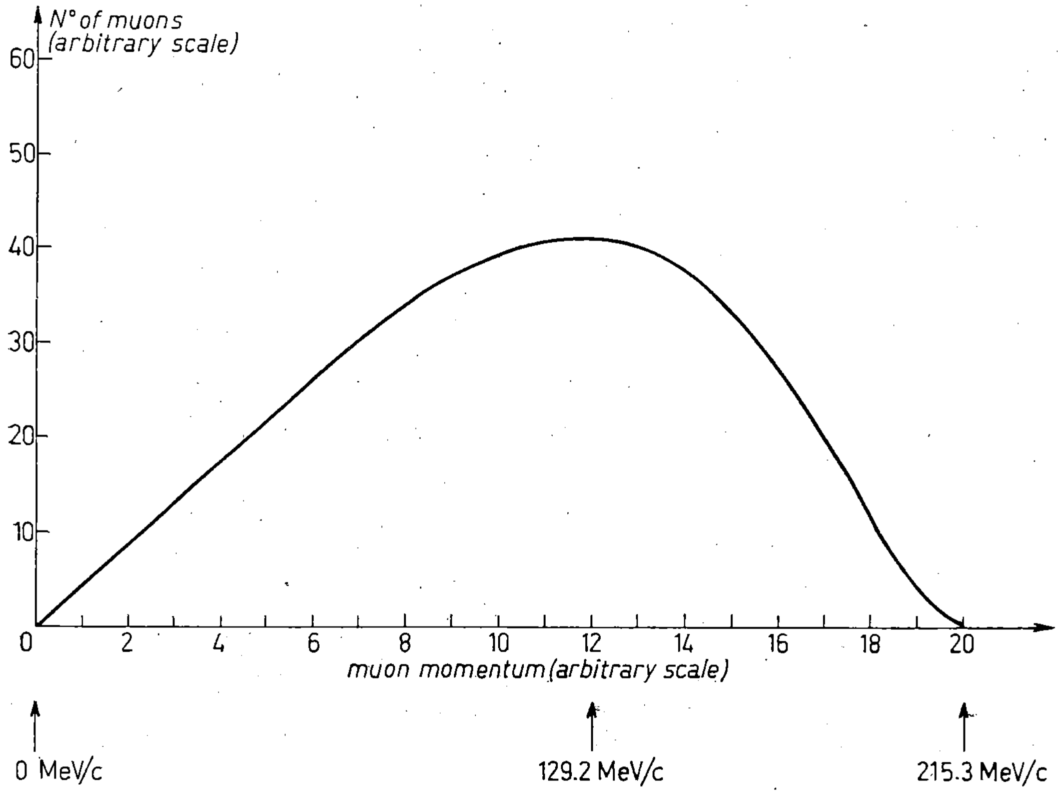

Fig. 3. – Muon spectrum in $K_{\mu 3}$-decay mode. The abscissa is linear in the muon momentum. In the energy scale the endpoint corresponds to muon total energy of 239.8 MeV and the peak to 166.9 MeV.

energy of 167 MeV, which reserves the possibility of distinguishing our model experimentally from the other models ([13]). The $K'K\pi$ coupling constant g is related to the full width of the K′-particle Γ by ([14])

$$\Gamma = w(K^{-\prime} \to \overline{K}^0 + \pi^-) = \frac{(\sqrt{2}g)^2}{4\pi} \cdot \frac{2\alpha^3}{3} \cdot m_{K'} \, ,$$

([13]) N. Brene, L. Egardt and B. Qvist: *Nucl. Phys.*, **22**, 553 (1961).
([14]) M. A. B. Bég and P. C. De Celles: *Phys. Rev. Lett.*, **6**, 145 (1961).

where α is the momentum of the final kaon or pion in units of $m_{K'}$. The experimental width 16 MeV [11] gives the estimate

$$\frac{g^2}{4\pi} \sim 0.83 .$$

The weak K'eν coupling constant f then will be found from the experimental decay rate of K_{e3}, $w(K_{e3}) = 3.36 \cdot 10^6 \text{ s}^{-1}$, as

$$\frac{f^2}{4\pi} \sim 6.7 \cdot 10^{-15} .$$

Inserting this value we obtain

$$w(\Lambda_\beta) = \frac{g_\Lambda^2}{4\pi} \times 4.1 \cdot 10^5 \text{ s}^{-1} ,$$

$$w(\Sigma_\beta) = \frac{(\sqrt{2} g_\Sigma)^2}{4\pi} \times 2.4 \cdot 10^6 \text{ s}^{-1} ,$$

or alternatively the form factor $f_1(0)$ of the Λ-hyperon becomes

$$|f_1(0)|^2 \sim \frac{g_\Lambda^2}{4\pi} \times 0.029 .$$

To obtain the experimental decay rate, $w(\Lambda_\beta) \sim 5 \cdot 10^6 \text{ s}^{-1}$, the coupling constant $g_\Lambda^2/4\pi$ must be of the order of 10. GOURDIN and RIMPAULT [15] analysed the production process $\pi + \mathcal{N} \to Y + K$ in the pole approximation and arrived at the estimate

$$\frac{g_\Lambda^2}{4\pi} \sim 1.8 ,$$

as well as at the assignment that the ΛΣ relative parity be odd. If we accept this estimate, the «reduced» decay rate of the Λ-hyperon becomes

$$|f_1(0)|^2 + 2.96 |g_1(0)|^2 \sim 0.052 + 0.043 \sim 0.09 ,$$

while experimentally it should be 0.35. If the Σ-hyperon has opposite parity to Λ our model gives

$$w(\Sigma_\beta) = \frac{(\sqrt{2} g_\Sigma)^2}{4\pi} \times 7.4 \cdot 10^6 \text{ s}^{-1} ;$$

---

(15) M. GOURDIN and M. RIMPAULT: preprint (University of Bordeaux, 1961).

hence assuming $g_\Lambda \sim g_\pi$ we obtain

$$w(\Sigma_\beta) \sim 2.8 \cdot 10^7 \text{ s}^{-1},$$

which is fortuitously close to the experimental finding $w(\Sigma_\beta) \sim 2 \cdot 10^7 \text{ s}^{-1}$.

It is hard to draw any definite conclusion out of this model because it depends on numerical estimates of very preliminary nature. The meson cloud effects alone could explain about $\frac{1}{4}$ of the observed β-decay rate of the hyperon. However, it would be more sensible at the present stage to accept the breakdown of the universality in the unrenormalized V—A scheme phenomenologically, allowing the possibility that the β-decay of the hyperon may be mediated by bosons partially.

* * *

The author would like to thank Professor R. GATTO for his discussions and comments.

Note added in proof.

Professor H. PRIMAKOFF called my attention that the experimental K' width 16 MeV was instrumental, hence the true physical width would be much smaller than that. If the physical width turns out to be 3 MeV, say, then our estimate of f^2 will be enhanced by a factor 5, which brings the computed β-decay rate of the Λ-hyperon up close to the observed one. If this is the case we regain the hope for the universal $V-A$ theory. I would like to express my sincere thanks to Professor H. PRIMAKOFF for his comment and to Professors C. FRONSDAL and A. HALSTEINSLID for their hospitality at the Bergen International School of Physics, where this paper was discussed.

RIASSUNTO (*)

Si calcola il rapporto di decadimento degli iperoni ed il rapporto di decadimento leptonico del kaone in una raffigurazione con un bosone intermedio, che è la particella K′. Si fa una stima numerica, di carattere preliminare, delle costanti di accoppiamento.

(*) *Traduzione a cura della Redazione.*